Killara HIGH SCHOOL

Student Number

2016

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Extension 1 Mathematics

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General Instructions

- Reading time – 5 minutes
- Working tine - 2 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question

Total Marks – 70

Section I - Pages ***

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages ****

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
Total	/70

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 40% of the Higher School Certificate Course Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for questions 1 – 10 (Detach from paper)

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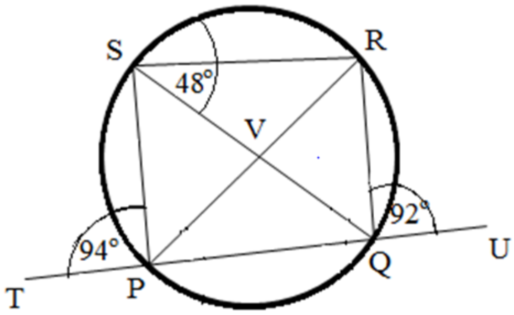
1)	In how many ways can 8 students be arranged if the tallest is first and the shortest is last? **(A)** ${}^{8}C_{6}$ **(B)** ${}^{6}C_{6}$ **(C)** ${}^{8}P_{6}$ **(D)** ${}^{6}P_{6}$
2)	If $x = 1.6$ is a close root of the equation $x^3 - 4x + 2 = 0$, find a better approximation to two decimal places. **(A)** 1.68 **(B)** 13.71 **(C)** 0.43 **(D)** 4.96
3)	Evaluate $\sin^{-1}\left(-\frac{1}{2}\right)$ as an exact answer. **(A)** $\frac{\pi}{6}$ **(B)** $\frac{5\pi}{6}$ **(C)** $-\frac{\pi}{6}$ **(D)** $-\frac{5\pi}{6}$

4) A cone has a base diameter of 16 cm and a perpendicular height of 12 cm. The angle the side of the cone makes with its base is:

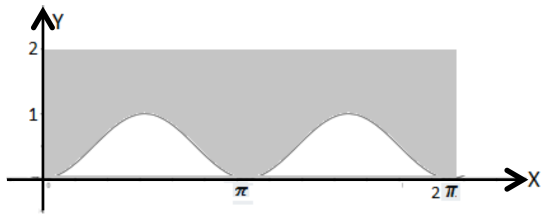
(A) 56°
(B) 37°
(C) 34°
(D) 53°

5) Given <TPS=94°, < RQU = 92°, < QSR = 48°, find <SVR



(A) 82°
(B) 92°
(C) 94°
(D) 96°

6)



The diagram shows a sketch of the curve $y = \sin^2 x$ between x=0 and x= 2π.
The shaded area equals:

(A) 2π square units
(B) 3π square units
(C) 2(2π − 1) square units
(D) 4π square units

7) The equation $\cos(2x) = \operatorname{cosec}\left(x - \frac{\pi}{2}\right)$ $-\pi \le x \le \pi$, has how many solutions?

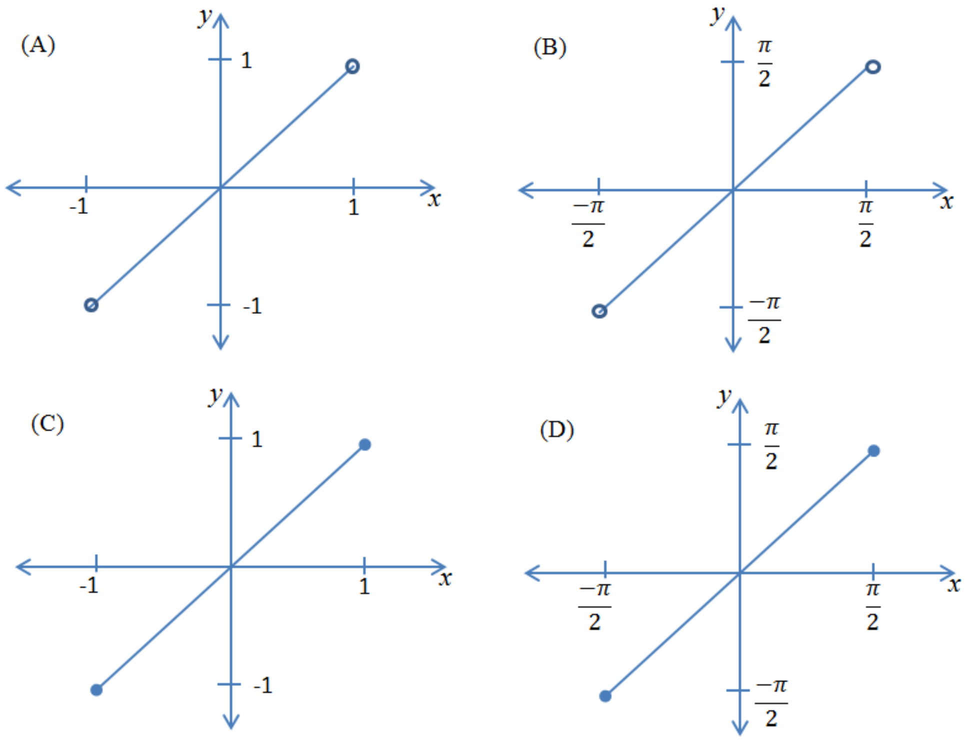
(A) 0

(B) 1

(C) 2

(D) 3

8) Which of the following is the graph of $\sin(\sin^{-1}x)$?



9) Using the substitution $u = \sqrt{x}$, $\int \frac{dx}{x+\sqrt{x}}$ can be transformed to:

(A) $\int \frac{2du}{u+1}$

(B) $\int \frac{du}{u^2+u}$

(C) $\int \frac{2du}{u^2+u}$

(D) $\frac{1}{2}\int \frac{du}{u^2+u}$

10)	The speed v m/s of a point moving along the x axis is given by $v^2 = 36 + 6x - 2x^2$, where x is in meters. The period and amplitude of the motion are: **(A)** Period π and amplitude $\frac{9}{2}$ **(B)** Period 2π and amplitude $\frac{\sqrt{63}}{2}$ **(C)** Period $\sqrt{2}\pi$ and amplitude $\frac{9}{2}$ **(D)** Period $\sqrt{2}\pi$ and amplitude $\frac{3}{2}$
11)	A particle undergoes linear acceleration according to the equation $a = (x + 4)^3\ m/s^2$. Given that the particle commences motion at the origin with a velocity 4 *m/s*, what is the particle's displacement when v = 10 *m/s*, given that $x < 0$? **(A)** -0.1313 **(B)** -7.7606 **(C)** -8.5378 **(D)** -20.0478

Section II

70 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Solve $\dfrac{3}{2x-1} < 2$ **3**

(b) Find $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{9+4x^2}$ giving your answer in exact form **3**

(c) Let A be the point (-2, 5) and B be the point (4, 1) **2**

Find the coordinates of the point P that divides the interval externally in the ratio 4: 3.

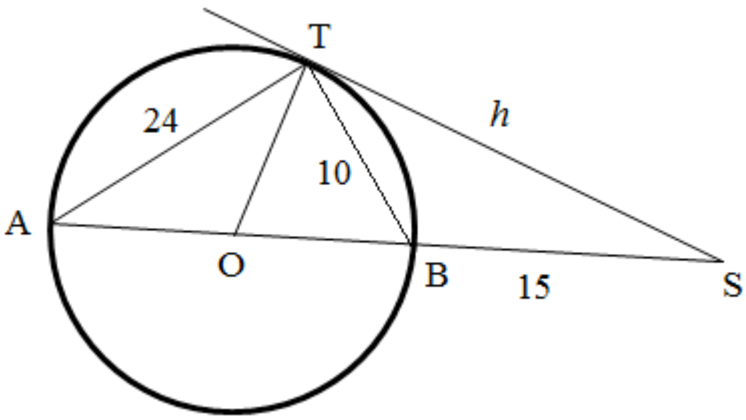
(d) Evaluate $\displaystyle\int \frac{x\,dx}{\sqrt{x-2}}$ using the substitution $u^2 = x-2$ **3**

(e) Evaluate $\displaystyle\lim_{x \to 0} \frac{\sin\left(\frac{x}{3}\right)}{2x}$ **2**

(f) Find $\dfrac{d}{dx}\left(e^{\sin^{-1}(3x)}\right)$ **2**

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) ST is a tangent at T, AT = 24cm, BT = 10cm, BS =15cm and ST = h cm. O is the centre of the circle. **2**



Find the value of h correct to one decimal place

(b) **i)** Express $\sqrt{3}\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < 2\pi$ **2**

ii) Hence or otherwise solve the equation $\sqrt{3}\cos\theta - \sin\theta = 1$, for $0 \le \theta \le 2\pi$ **2**

(c) Solve the equation $x^3 - 21x^2 + 126x - 216 = 0$ given that the three roots form a geometric series **3**

(d) The acceleration of a particle moving in a straight line is given by $\frac{d^2x}{dt^2} = -2e^{-x}$ where x is the displacement (in metres) from the origin. Initially the particle is at the origin and is moving with a velocity of $2ms^{-1}$

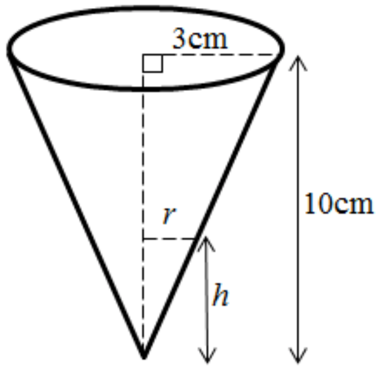
i) Prove that $v = 2e^{-\frac{x}{2}}$ **2**

ii) Find an expression for the displacement x at any time t. **2**

Question 12 continues on page 8

Question 12 (continued)

(e) The diagram shows a conical wheat flue. The flue is being filled at the rate of $2m^3$ / minute. The height of wheat at any time, '*t*' minutes, is '*h*' metres, and the radius of the wheat's top surface is 'r' metres.



i) Show that $r = \dfrac{3h}{10}$ **1**

ii) Find the rate at which the height is increasing when the height of wheat is 8 *m* **3**

(The volume of a cone is given by $V = \dfrac{1}{3}\pi r^2 h$

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) Consider the function $f(x) = 2\sin^{-1}(x-1)$.

iii) Find the domain and range of the function. **1**

iv) Sketch the graph of the function. **1**

v) Find the equation of the inverse function. **2**

(b) Four people go to a pizza festival, where four different gourmet pizzas A, B, C and D are offered. Each person chooses a pizza at random to try.

i) Find the probability that they all choose different pizzas. **2**

ii) Find the probability that exactly two of the people choose pizza A. **2**

(c) **i)** $T(2at,\ at^2)$ is a point on the parabola $x^2 = 4ay$. Show that the normal to the parabola at T has equation $x + ty - 2at - at^3 = 0$. **2**

ii) P and Q are points on the parabola $x^2 = 4ay$ with parameter values $t = 1$ and $t = 2$ respectively. Show that the normal to the parabola at P and Q intersect at a point R on the parabola. **2**

(d) The formula for the nth term a_n of the Fibonacci sequence, **3**

$$1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

is given by,

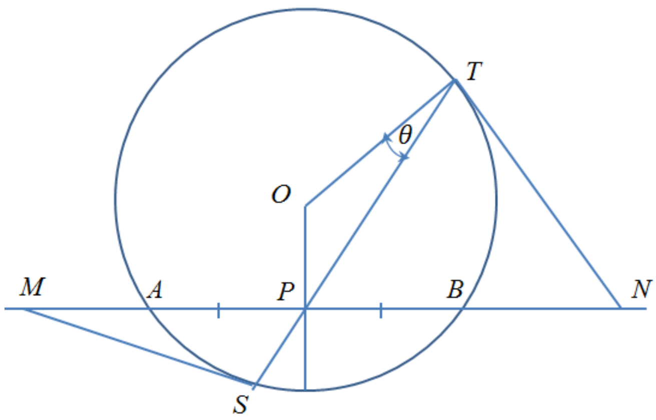
$$a_n = \begin{cases} 1 \text{ for } n = 1 \text{ and } 2 \\ a_{n-2} + a_{n-1} \text{ for } n > 2 \end{cases}$$

Prove by mathematical induction that,

$$a_n = \frac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{\sqrt{5}\,2^n}$$

Question 14 (15 marks) Use a SEPARATE writing booklet

(a)



In the diagram above, P is the midpoint of the chord AB in the circle with centre O. A second chord ST passes through P, and the tangents at the endpoints meet, AB produced at M and N respectively.

Join OS.

i) Explain why $OPNT$ is a cyclic quadrilateral. **2**

ii) Explain why $OPSM$ is also cyclic. **1**

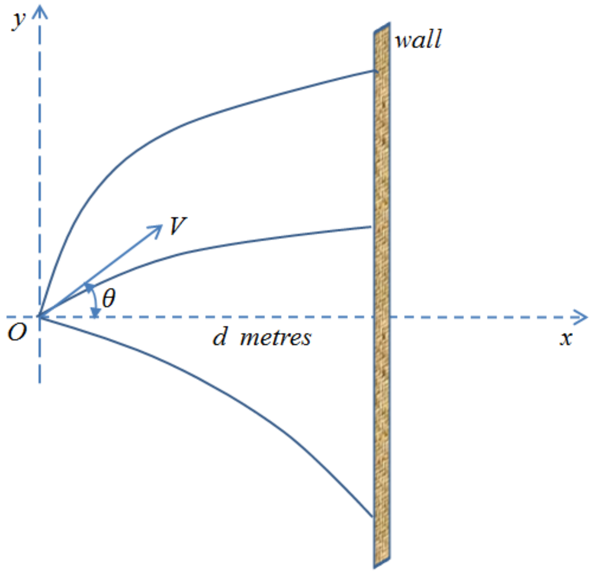
iii) Let $\angle OTS = \theta$. Show that $\angle ONP = \angle OMP = \theta$. **2**

iv) Hence, prove that $AM = BN$. **1**

Question 14 continues on page 11

Question 14 (continued)

(b)



In the diagram above, a large number of projectiles are fired simultaneously from O, each with the same velocity V m/s, but different angles of projection θ, at a wall d meters from O. The projectiles are fired so that they all lie in the same vertical plane perpendicular to the wall.

You may assume that the equations of motion at time t are given by:

$$x = Vt\cos\theta \quad \text{and} \quad y = -\frac{1}{2}gt^2 + Vt\sin\theta.$$

i) Using these two equations of motion, prove that the relationship between the height y and time t is: **2**

$$4y^2 + 4gt^2y + \left(g^2t^4 + 4x^2 - 4v^2t^2\right) = 0.$$

ii) Show that the first impact at the wall occurs at time $t = \frac{d}{V}$ and that this projectile was fired horizontally. **2**

iii) Hence, find where this projectile hits the wall. **1**

iv) Show that for $t > \frac{d}{V}$, there are *two* impacts at time t, and that the distance between these is: **2**

$$2\sqrt{V^2t^2 - d^2}.$$

v) Given that $V = 10$ m/s and $d = 10$ metres, what are the initial angles of projection of the two projectiles that will strike the wall simultaneously $20\sqrt{3}$ metres apart. **2**

End of Examination ☺

9

Multiple choices

(1) I - - - - - - - - S (D)

$^6P_6$

(2) $f'(x) = 3x^2 - 4$

$f'(1.6) = 3.68$

$f(1.6) = -.304$ (A)

$x_2 = 1.6 - \frac{-.304}{3.68} = 1.68$

(3)

$\sin^{-1}\left(\frac{-1}{2}\right) = -\pi/6$ (C)

(4) $y = \frac{e^x - 2}{e^x + 2} = \frac{e^x + 2 - 4}{e^x + 2} = 1 - \frac{4}{e^x + 2}$ (B)

when $x \to \infty$, $y \to 1$

$x \to -\infty$ $y \to 1 - \frac{4}{0+2} = -1$

(5) (A)

(6)

$A = 2 \times 2\pi - \int_0^{2\pi} \sin^2 x$

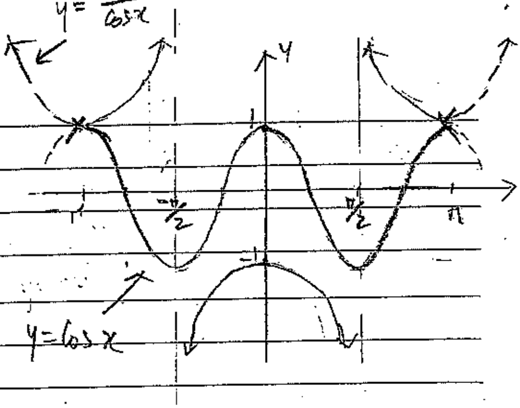
$= 4\pi - \frac{1}{2}\int_0^{2\pi} 1 - \cos 2x \, dx$ (B)

$= 4\pi - \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{2\pi}$

$= 3\pi$

(7) $\cos 2x = -\dfrac{1}{\sin(\frac{\pi}{2}-x)}$

$= \dfrac{-1}{\cos x}$



from the graph, there are 2 solutions or

solve $\cos 2x = -\dfrac{1}{\cos x}$

$2\cos^3 x - \cos x + 1 = 0$

$\Rightarrow x = \pm \pi$

(8)

$\sin(\sin^{-1} x) = x$

domain $-1 \le x \le 1$

Range $-1 \le 1 \le 1$

(a)

(9)

$u = \sqrt{x}$

$du = \dfrac{1}{2\sqrt{x}} dx \Rightarrow dx = 2u\,du$

$\displaystyle\int \frac{dx}{x+\sqrt{x}} = \int \frac{2u\,du}{u^2+u} = 2\int \frac{du}{u+1}$

(A)

(10)

$\dfrac{d}{dx} \dfrac{1}{2} v^2 = \dfrac{d}{dx}\, 18 + 3x - x^2$

$= 3 - 2x = 2\left(\frac{3}{2} - x\right)$

(C)

$n = \sqrt{2}$ $T = \dfrac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

$x^2 - 3x - 18 = 0$ $x = 6$ $a = \dfrac{9}{2}$

$(x-6)(x+3) = 0$ $x = -3$

Q11

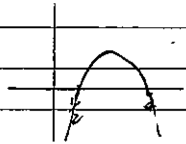
a) $\frac{3}{2x-1} < 2$

$3(2x-1) - 2(2x-1)^2 < 0$

$(2x-1)(3-4x+2) < 0$

$(2x-1)(5-4x) < 0$

$x < \frac{1}{2}$ and $x > \frac{5}{4}$



✓ ✓ ✓

generally well done

b) $\int_0^{\sqrt{3}/2} \frac{dx}{9+4x^2} = \int_0^{\sqrt{3}/2} \frac{dx}{3^2+(2x)^2}$

$= \frac{1}{6}\left[\tan^{-1}\frac{2x}{3}\right]_0^{\sqrt{3}/2}$

$= \frac{1}{6} \times \frac{\pi}{6} = \frac{\pi}{36}$

✓ ✓ ✓

generally well done

c) $x = \frac{-4\times 4 + 3\times -2}{-1} = 22$

$y = \frac{-4\times 1 + 3\times 5}{-1} = -11$

$(22, -11)$

✓ ✓

generally well done

11 cont.

(d) $u^2 = x-2 \Rightarrow x = u^2+2$		
$2u\,du = dx$		generally well done
$\int \frac{x\,dx}{\sqrt{x-2}} = \int \frac{u^2+2}{u} \cdot 2u\,du$	✓	
$= \int 2u^2 + 4\,du$		
$= \frac{2}{3}u^3 + 4u + C$	✓	few students did not express the answer in term of x.
$= \frac{2}{3}(x-2)^{3/2} + 4(x-2) + C$	✓	
e) $\lim_{x \to 0} \frac{\sin x/3}{x/3} \times \frac{1/3}{2} = 1 \times \frac{1}{6}$	✓ ✓ Working must be shown correctly to obtain full marks.	generally well done
$= \frac{1}{6}$		
f) $\frac{d}{dx} e^{\sin^{-1}(3x)} = e^{\sin^{-1}(3x)} \cdot \frac{3}{\sqrt{1-9x^2}}$	✓ ✓	generally well done
$= \frac{3e^{\sin^{-1}(3x)}}{\sqrt{1-9x^2}}$		

Q.12

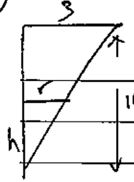
a) $R = \sqrt{3+1} = 2$	✓	generally well done
$\alpha = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$	✓	
$\therefore \sqrt{3}\cos\theta - \sin\theta = 2\cos\left(\theta + \frac{\pi}{6}\right)$		
$2\cos\left(\theta + \frac{\pi}{6}\right) = 1$		generally well done
$\theta + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}$	✓	
$\therefore \theta = \frac{\pi}{6}$ and $\frac{3\pi}{2}$	✓	
c) let the 3 roots be $\frac{a}{r}$, a and ar		
Σ roots $= \frac{a}{r} + a + ar = 21$ ①		• few students used polynomial division to find the answers.
Prod of roots $\frac{a}{r} \cdot a \cdot ar = 216$		
$a^3 = 216$		
$a = 6$ Sub → ①	✓ Correct value of a	• few students had no idea how to approach this problem.
$\Rightarrow \frac{6}{r} + 6 + 6r = 21$		
$6r^2 - 15r + 6 = 0$		
$(2r-1)(r-2) = 0$	✓ Correct working	
$\Rightarrow r = \frac{1}{2}$ or 2		
$\therefore$ The 3 roots are 3, 6 and 12	✓ Correct answers	

12 : cont

c) $\frac{d}{dx}\frac{1}{2}v^2 = -2e^{-x}$		
$\frac{1}{2}v^2 = 2e^{-x} + c$	✓ Correct expression of $\frac{1}{2}v^2$	
when $x = 0$, $v = 2$		
$\Rightarrow c = 2 - 2 = 0$		many students assumed
$\therefore \frac{1}{2}v^2 = 2e^{-x}$		that $v = 2e^{-x/2}$ without
$v^2 = 4e^{-x}$	✓ Correct answer of v^2	giving reason.
$v = \pm 2e^{-x/2}$		
when $t = 0$, $x = 0$, $v = 2\text{m/s} > 0$	✓ final answer with reason	
$\therefore v = 2e^{-x/2}$		
(ii) $\frac{dx}{dt} = 2e^{-x/2}$		
$dt = \frac{1}{2}e^{x/2}$		generally well done.
$\Rightarrow t = e^{x/2} + c$	✓ Correct expression for t	
when $t = 0$, $x = 0 \Rightarrow c = -1$		
$\therefore e^{x/2} = t + 1$		
hence $x = 2\ln(t+1)$		
or $= \ln(t+1)^2$	✓ Correct answer	

Q12 cont

e)



using similar $\triangle$s' $\frac{r}{3} = \frac{h}{10}$ $\Rightarrow r = \frac{3h}{10}$	✓	well done
ii) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3h}{10}\right)^2 h$ $= \frac{3\pi h^3}{100}$		• many students finding $\frac{d}{dh}\frac{1}{3}\pi r^2 h$ without
$\frac{dV}{dh} = \frac{9\pi h^2}{100}$	✓ Correct expression for $\frac{dV}{dh}$	sub. $h = \frac{3h}{10}$ before hand. This gave incorrect answer for $\frac{dV}{dh}$.
$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ where $\frac{dV}{dt} = 2\,cm^3/min.$ $= \frac{100}{9\pi h^2} \times 2 = \frac{200}{9\pi h^2}$	✓ Correct expression for $\frac{dh}{dt}$	
Sub $h = 8$ $\Rightarrow \frac{dh}{dt} = \frac{200}{9 \times \pi \times 8^2}$ $= \frac{25}{72\pi}$ cm $= 0.11$ cm	✓ Correct substitution & answer.	• Some did the long way using $\frac{dh}{dt} = \frac{dV}{dr} \times \frac{dr}{dh} \times \frac{dh}{dV}$ and find these derivative functions.

(a) Consider the function $f(x) = 2\sin^{-1}(x-1)$.

i) Find the domain and range of the function. **1**

Criteria	Marks
• Provides a correct solution	1

Sample answer:

Domain: $\{-1 \le x-1 \le 1\}$

: $\{0 \le x \le 2\}$

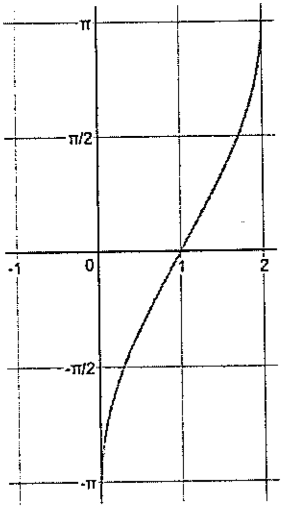
Range: $\{-\pi \le y \le \pi\}$

Mostly done well

ii) Sketch the graph of the function. **2**

Criteria	Marks
• Draws a correct sketch	2
• Draws a graph showing the correct shape, or equivalent merit	1

Sample solution:



be careful the shape is right.

(2,π) is a horizontal tangent

	iii) Find the equation of the inverse function.	2

Criteria	Marks
• Provides a correct solution	2
• Swaps variables and makes some progress towards a correct solution, or equivalent merit.	1

Sample solutions:

Swap unknowns

$$y = 2\sin^{-1}(x-1)$$

$$\frac{1}{2}y = \sin^{-1}(x-1)$$

$$\sin\left(\frac{y}{2}\right) = x-1$$

$$x = 1+\sin\left(\frac{y}{2}\right)$$

hence

$$f^{-1}(x) = 1+\sin\left(\frac{x}{2}\right) \text{ for } -\pi \le x \le \pi$$

well done

(b) Four people go to a pizza festival, where four different gourmet pizzas A, B, C and D are offered. Each person chooses a pizza at random to try.

	i) Find the probability that they all choose different pizzas.	1

Criteria	Marks
• Provides a correct solution	1

Sample solution:

$$P(\text{all different}) = \frac{4!}{4^4}$$

$$= \frac{3}{32}$$

	ii) Find the probability that exactly two of the people choose pizza A.	2

Criteria	Marks
• Provides a correct solution	2
• Makes progress towards an answer with at most one option missed or one element unconsidered, or equivalent merit.	1

Sample solution:

$$P(\text{exactly 2 choose A}) = {}^4C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2$$

$$= \frac{27}{128}$$

make sure to show working

(c)	P and Q are points on the parabola $x^2 = 4ay$ with parameter values $t = 1$ and $t = 2$ respectively.	
	i) Given the equation of the normal at T is $x + ty - 2at - at^3 = 0$ (Do not prove). Find the equation of the normal at P and at Q.	1

Criteria	Marks
• Provides a correct solution	1

Sample solution:

when t is 1 the normal at P:

$$x + 1.y - 2.a.1 - a.1^3 = 0$$
$$x + y = 3a$$

when t is 2 the normal at Q:

$$x + 2.y - 2.a.2 - a.2^3 = 0$$
$$x + 2y = 12a$$

Many students confused p & q as variables instead of substituting in 1 & 2

	ii) Show that the normal to the parabola at P and Q intersect at a point R on the parabola.	2

Criteria	Marks
• Provides a correct solution	2
• Solve the normal equations simultaneously and attempt to demonstrate solution lies on the parabola, or equivalent merit	1

Sample solution:

$$x + y = 3a \text{---} 1$$
$$x + 2y = 12a \text{--} 2$$

subtracting 2 from 1

$$-y = -9a$$
$$y = 9a$$

substituting into 1

$$x + 9a = 3a$$
$$x = -6a$$

giving $R(-6a, 9a)$ as the point of intersection

$$x^2 = 36a^2$$
$$= 4a(9a)$$
$$= 4ay$$

$\therefore$ R lies on the parabola

Some students forgot to show the point lies on the parabola

(d) i) Prove the following by the process of mathematical induction. **3**

$$\ln(2)+\ln\left(\frac{3}{2}\right)+\ln\left(\frac{4}{3}\right)+...+\ln\left(\frac{n+1}{n}\right)=\ln(n+1)$$

Criteria	Marks
• Provides a correct solution	3
• Demonstrate that k implies k+1	2
• Demonstrate the first case	1

Sample solution:

when $n=1$

$$\ln\left(\frac{1+1}{1}\right)=\ln(2)$$

$$=\ln(1+1)$$

$\therefore$ statement true for $n=1$

Assume true for $n=k$

that is:

$$\ln(2)+\ln\left(\frac{3}{2}\right)+\ln\left(\frac{4}{3}\right)+...+\ln\left(\frac{k+1}{k}\right)=\ln(k+1)$$

when $n=k+1$

$$\ln(2)+\ln\left(\frac{3}{2}\right)+\ln\left(\frac{4}{3}\right)+...+\ln\left(\frac{k+1}{k}\right)+\ln\left(\frac{k+1+1}{k+1}\right)$$

$$=\ln(k+1)+\ln\left(\frac{k+1+1}{k+1}\right)$$

$$=\ln((k+1)+1)$$

as required

$\therefore$ as n=1 is true and n=k true proves n=k+1 is true

the statement is true by the principle of mathematical induction.

Some students made logically false statements. learn the process of induction

ii) Hence find p for which $\sum_{n=1}^{p}\ln\left(\frac{n+1}{n}\right)\geq\pi$ **1**

Criteria	Marks
• Provides a correct solution	1

Sample solution:

sum of

$$\sum_{n=1}^{p}\ln\left(\frac{n+1}{n}\right)\geq\pi$$

$$\ln(p+1)\geq\pi$$

$$p+1\geq e^{\pi}$$

$$p\geq e^{\pi}-1$$

$$p\geq 22.14...$$

$$\therefore p=23$$

mostly done well

14a) (i)	From the equations of motion, $V\cos\theta = \frac{x}{t}$ $V\sin\theta = \frac{y}{t} + \frac{1}{2}gt$ $\left(\frac{x}{t}\right)^2 + \left(\frac{y}{t} + \frac{1}{2}gt\right)^2 = V^2(\cos^2\theta + \sin^2\theta)$ $(x)^2 + \left(y + \frac{1}{2}gt^2\right)^2 = V^2t^2$ $x^2 + y^2 + ygt^2 + \frac{1}{4}g^2t^4 = V^2t^2$ $4y^2 + 4ygt^2 + (4x^2 + g^2t^4 - 4V^2t^2) = 0$	2 marks: correct proof 1 mark: substantial progress is made in trying to eliminate $\sin\theta$ and $\cos\theta$	*Students who could understand, "eliminate θ was the purpose of the question did it correctly. Well done by many students*
(ii)	$t = \frac{x}{V\cos\theta}$ where $x = d$ and V are constants. Hence, t is minimum when $\cos\theta$ is maximum which occurs when $\theta = 0$. ie. The projectile is fired horizontally. **1 mark** $\theta = 0$, then $\cos\theta = 1$. Hence, $t = \frac{x}{V\cos\theta} = \frac{d}{V}$ seconds. **1 mark**	1 mark: proves angle of projection $\theta = 0$. 1 mark: proves the result with correct working and reasoning.	*Very few students realised that "first" impact means, you need to minimise t.* *Most of the students <u>used</u> $\theta = 0$ to prove $t = \frac{d}{V}$ and vice versa. These students were awarded 0 marks. They have not proved any of the required results.*
(iii)	$y = -\frac{1}{2}gt^2 + Vt\sin\theta$ $\theta = 0$, then $\sin\theta = 0$. $y = -\frac{1}{2}g\left(\frac{d}{V}\right)^2 = -\frac{gd^2}{2V^2}$ (hits below the horizontal.)	1 mark: substitutes $\theta = 0$ into the equation and gives the correct result.	*Most students got this mark, except those who did not realise $\theta = 0$.*
(iv)	To have two impacts on the wall, there need to prove that here are two solutions for y when $t > \frac{d}{V}$. $4y^2 + 4gt^2y + (g^2t^4 + 4x^2 - 4V^2t^2) = 0$ $x = d$, $\Delta = (4gt^2)^2 - 4 \times 4(g^2t^4 + 4d^2 - 4V^2t^2)$ $= 16g^2t^4 - 16(g^2t^4 + 4d^2 - 4V^2t^2)$ $= 64(V^2t^2 - d^2)$. When $t > \frac{d}{V}$, $Vt > d$. Hence, $V^2t^2 - d^2 > 0$, and $\Delta > 0$. Ie. There are two solutions for y. This means that there are two real and	1 mark: proves $\Delta > 0$ for the quadratic in y.	*Many students did not attempt this question. Those who realised two impacts at given t meant that there are two values for y.* *Majority of the students who attempted this question got 1 mark for setting $\Delta > 0$.*

	distinct roots for y and so, there are two impacts at the same t. **1 mark.** Now the distance between the impacts equals the difference between the roots. The roots are $\frac{-b\pm\sqrt{\Delta}}{2a}$ Hence the difference = $\frac{2\sqrt{\Delta}}{2a}$ = $\frac{\sqrt{\Delta}}{a}$ $= \frac{\sqrt{64\left(V^2t^2-d^2\right)}}{4}$ $= 2\sqrt{\left(V^2t^2-d^2\right)}$ **1 mark** **Another approach:** $y_1+y_2=-gt^2$ $y_1y_2=\frac{g^2t^4+4x^2-4v^2t^2}{4}$ $y_1+y_2=\sqrt{(y_1+y_2)^2-4y_1y_2}$ $=\sqrt{g^2t^4-g^2t^4-4x^2+4v^2t^2}$ $=2\sqrt{v^2t^2-x^2}$ $=2\sqrt{v^2t^2-d^2}$	1 mark: correct answer from correct working	
(v)	Distance = $2\sqrt{\left(V^2t^2-d^2\right)}$ $= 2\sqrt{\left(100t^2-100\right)}$ $= 20\sqrt{\left(t^2-1\right)}$ $20\sqrt{\left(t^2-1\right)} = 20\sqrt{3}$ $t^2-1=3$ $t=2,\quad t>0$ $\cos\theta=\frac{x}{Vt}=\frac{10}{10\times 2}=\frac{1}{2}$ Hence, $\theta = 60°$ and $-60°$ are the angles of projection.		*Students who realised this question is a follow up of (iv) got the mark.* *Many set* $y=20\sqrt{3}$ in $y=V\sin\theta t-\frac{1}{2}gt^2$ *and tried to solve for t unsuccessfully.*

14b) (i)	$\angle OTN = 90°$ ($OT \perp TN$, angle between tangent and radius is $90°$ $\angle OPN = 90°$ (the line from the centre that **bisects the chord** is perpendicular to the chord) **1 mark** *OPNT* is cyclic as the opposite angles add to $180°$ **1 mark**	1 mark: both reasoning statements are correct 1 mark: gives the reason for the quadrilateral to be cyclic.	*Well done.* *A very common error the statement:* ***Line from the centre to the tangent makes*** $90°$.  ***Cleary the diagram above doesn't.*** *Also, as shown in the answers, only if the chord is **bisected**, it is perpendicular.* ***Note: you need to learn and present the statements of the theorems accurately.***
(ii)	Similarly, $\angle OPM = 90°$ (from (i) $OP \perp PN$ and hence *MN*.) $\angle OSM = 90°$ (angle between tangent and radius is $90°$ *OPSM* is cyclic as the angles subtended by arc OM in the same segment are equal.) **1 mark**	1 mark: gives the correct reasoning for the quadrilateral to be cyclic.	
(iii)	$\angle OTS = \angle OST = \theta$ (OT = OS radii, angles opposite equal sides of isosceles ΔOTS **1 mark** Now, $\angle OTP = \angle ONP = \theta$ and $\angle OSP = \angle OMP = \theta$ angles in the same segment of cyclic quadrilaterals OTNP and OPSM. **1 mark** Hence, $\angle ONP = \angle OMP = \theta$.	2 marks: both reasoning correct 1 mark only: if substantial progress	*A common error was in the interpretation of the diagram. Some students misunderstood the notation on the diagram meant MP = PN, rather than AP = PB. You always need to verify the diagram from the description of facts stated in the question.*
(iv)	ΔOMN is isosceles (OM = MN as $\angle ONP = \angle OMP = \theta$. sides opposite equal angles are equal) OP is the altitude. Hence, MP = PN (perpendicular bisects the opposite side) Hence, $MP - AP = PN - PB$ (equals subtracted from equals) Ie. AM = BN	1 mark: correct reasoning	*Well done*